# MPC Planner for UAV Trajectory Tracking and Obstacle Avoidance

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## Introduction

### *Motivation*

- Quadrotors have become increasingly popular in navigating tight spaces or indoors [1].
- Dynamic obstacle avoidance allows higher robustness against moving obstacles and/or imperfect sensing and localization [2]
- MPC can offer superior computational efficiency and generalizability, and dynamic feasibility when compared to reinforcement learning [3], and planning-based methods [2], respectively.

### *Goal*

- Design an MPC planner to track any arbitrary trajectory and form a collision-free path from static and dynamic obstacles

## Methodology

### *System Dynamics*

A linear dynamic model is used to reduce the MPC Complexity. [4]

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, \text{where } x = \begin{bmatrix} p \\ v \end{bmatrix}, p = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix}, v = \begin{bmatrix} \dot{P}_x \\ \dot{P}_y \\ \dot{P}_z \end{bmatrix}, \text{and } u = [a], a = \begin{bmatrix} \ddot{P}_x \\ \ddot{P}_y \\ \ddot{P}_z \end{bmatrix}$$

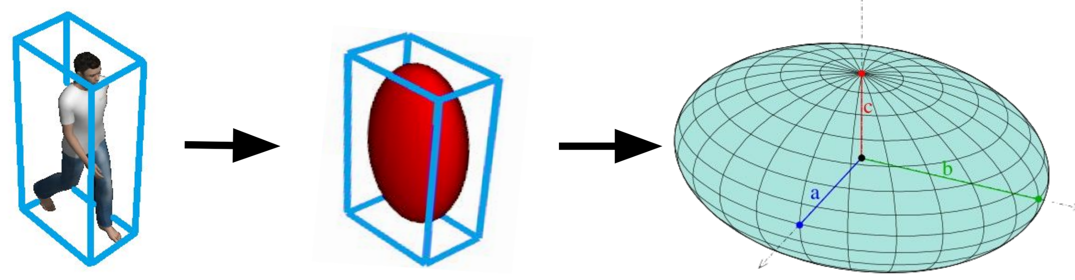
### *MPC Formulation*

Minimize the reference tracking error and control effort, subject to quadrotor state, dynamic constraint, kinodynamic constraint and obstacle contriant.

$$\min_{x_{1:N}, u_{1:N-1}} \sum_{i=1}^{N-1} \left[ \frac{1}{2}(x_i - x_{ref,i})^T Q (x_i - x_{ref,i}) + \frac{1}{2}(u_i - u_{ref,i})^T R (u_i - u_{ref,i}) \right] + \frac{1}{2}(x_N - x_{ref,N})^T Q_f (x_N - x_{ref,N})$$

$$\begin{aligned} \text{s.t.} \quad & x_1 = x_{\text{IC}} \\ & x_{i+1} = f(x_k, u_k) \quad \text{for } k = 1, 2, \ldots, N-1 \\ & x_{min} \le x_k \le x_{max} \\ & u_{min} \le u_k \le u_{max} \\ & \left(\frac{P_x - O_x}{a}\right)^2 + \left(\frac{P_y - O_y}{b}\right)^2 + \left(\frac{P_z - O_z}{c}\right)^2 \ge 1 \end{aligned}$$

Ellipsoid obstacle constraint is used in this MPC formulation.

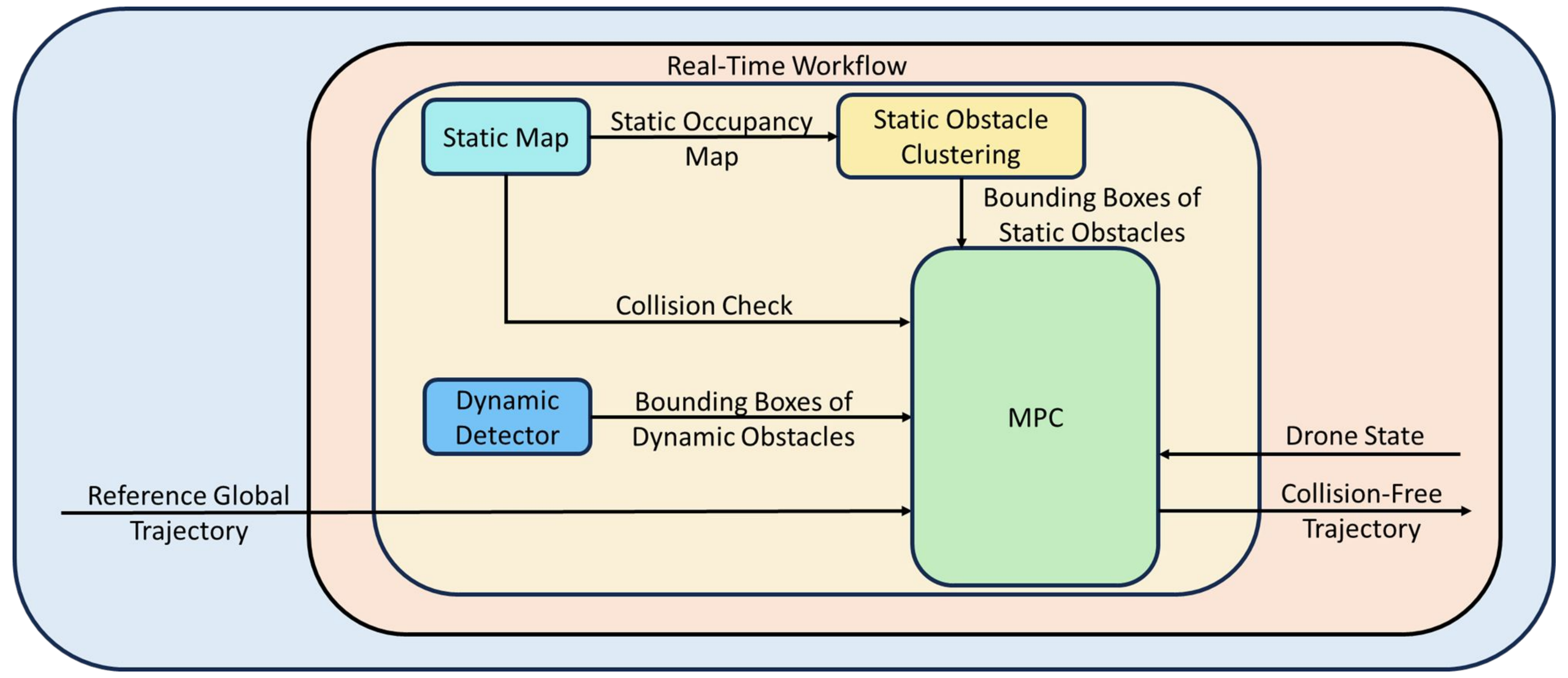


$$\left(\frac{P_x - O_x}{a}\right)^2 + \left(\frac{P_y - O_y}{b}\right)^2 + \left(\frac{P_z - O_z}{c}\right)^2 \ge 1$$

$P_x, P_y, P_z$ – X,Y,Z position of the drone
$O_x, O_y, O_z$ – X,Y,Z position of the obstacle
$a, b, c$ – X,Y,Z size of the obsatcle

## MPC Planner Framework

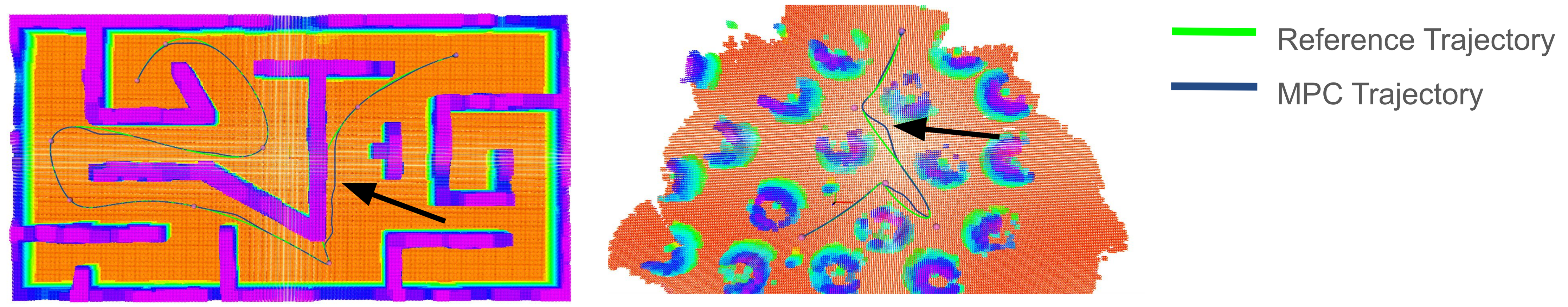
- Generate a polynomial-based reference trajectory with velocity, acceleration and corridor constraint.
- A Static obstacle clustering module pre-processes the static map into bounding boxes.
- The real-time MPC planner module tracks the reference and keep a safe distance to any obstacles.
- Map module provides a collision check for the trajectory generated by MPC.




## Result

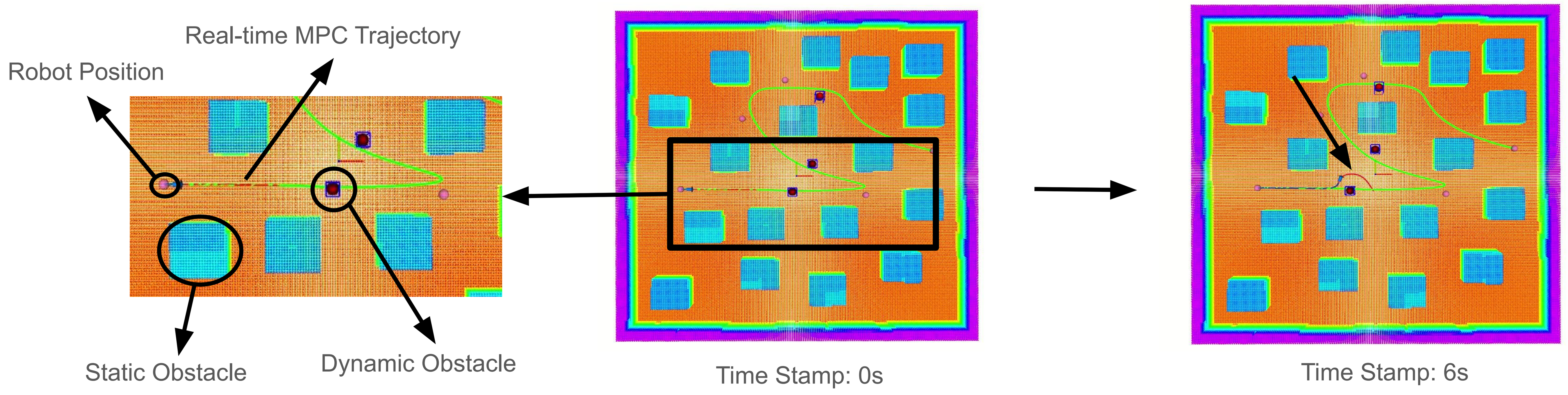
### *Static Obstacle Avoidance - Tracking a polynomial-based reference trajectory in different static environments*

The results show then the polynomial-based trajectory generator fails to solve for a 100% safe trajectory, MPC can avoid the static obstacles and keep a safe distance.




### *Dynamic Obstacle Avoidance - Avoiding moving objects by considering both dynamic and static obstacles*

The result shows the screenshot of two time step in one video. When a dynamic obstacle appears in the reference trajectory, by applying both static obstacles and dynamic obstacles constraints, a safe trajectory is generated.




## Conclusion

- A MPC formulation with linear dynamics and quadratic constraints for obstacle avoidance
- A MPC framework that generate safe trajectory in dynamic environment
- Effective and efficient system which can work on limited computational resource platform

## Future Work

- Dynamic obstacles motion prediction: take advantage of the predictive nature of MPC for collision avoidance
- Semantic map: use semantic information to help obstacle identification and motion prediction
- Improve MPC solving time: achieve better real-time performance

## References

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